SENTIFIC SECTION

SECONDARY 2

PURE MATHIMATICS

Geel 2000 Language Schools

Math Department

First Term

2022/2023

Name

.....

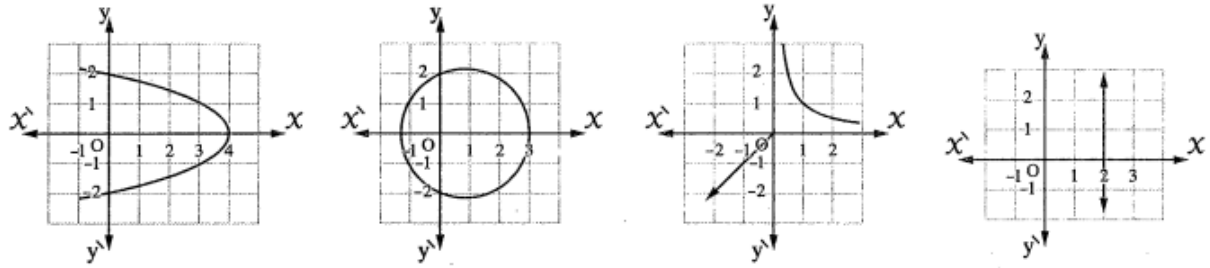
Class.....

(Unit 1) Real functions

(Domain, Range and monotony)

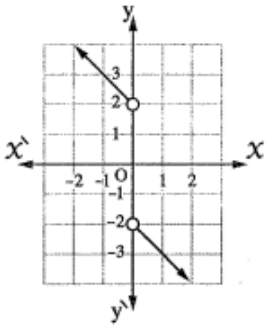
1-

(**1**) Which of the following figures represents a function of X ?



(a) (b) (c) (d)

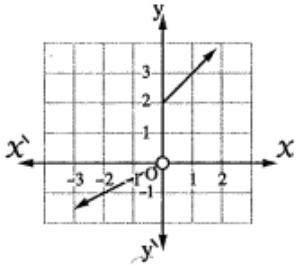
(**2**) The opposite figure represents a function of X whose domain is .....



(a) $\mathbb{R}$ (b) $\mathbb{R} -]-1 , 2[$

(c) $\mathbb{R} - [-1 , 2]$ (d) $\mathbb{R} - \{0\}$

(**3**) The opposite figure represents a function of X whose range is .....



(a) $\mathbb{R} - [0 , 2]$ (b) $\mathbb{R} - \{0\}$

(c) $\mathbb{R} - [0 , 2[$ (d) $\mathbb{R} -]0 , 2]$

(**4**) $f(X) = \sqrt{4 - X^2}$, then the domain of the function f = ...

(a) $[-2 , 2]$ (b) $]-2 , 2[$

(c) $[-2 , 2[$ (d) $]-2 , 2]$

Operations on functions (composition functions)

2-

Choose the correct answer from those given :

(**1**) $f(x)=\frac{1}{x}$, $g(x)=\sqrt{x}$, then the domain of $(f.g)$ = ...

(a) $\mathbb{R}-\{0\}$ (b) $\mathbb{R}$ (c) $\mathbb{R}^+$ (d) $[0,\infty[$

(**2**) $f(x)=x+1$, $g(x)=x^2$, then $(f\circ g)(2)$ = ...

(a) 3 (b) 4 (c) 5 (d) 9

(**3**) The domain of the function f : $f(x)=\sqrt{5-x}$ equals .....

(a) $\mathbb{R}-\{5\}$ (b) $\mathbb{R}^+$ (c) $]-\infty,5]$ (d) $[5,\infty[$

(**4**) $f(x)=\sqrt{x}$, $g(x)=x^2$, then the domain of $(f\circ g)$ = ...

(a) $[0,\infty[$ (b) $\mathbb{R}$ (c) $\mathbb{R}^+$ (d) $\mathbb{R}^-$

3-

If $f(x)=\frac{1}{x}$, $g(x)=x+3$, find :

(**1**) $(f\circ g)(x)$

(**2**) $(g\circ f)(x)$

and state the domain in each case.

4-

If $f(x)=\frac{1}{x}$, $g(x)=x+3$, find :

(**1**) $(f\circ g)(x)$

(**2**) $(g\circ f)(x)$

and state the domain in each case.

5-

If $f(X) = \frac{1}{X}$, $g(X) = 2X$, find each of the following :

(1) $(f + g)(X)$ (2) $(f . g)(X)$ (3) $\left(\frac{g}{f}\right)(X)$

Showing the domain.

Properties of functions

6-

Find the type of each function whether it is even , odd or otherwise :

(1) $f(x) = \dfrac{x^3}{|x|+2}$ **(2)** $f(x) = \sin x^2 - \sin^2 x$

7-

[b] If $f(x) = x - 1$, $g(x) = \sqrt{x}$, then find $(g \circ f)(x)$ and determine its domain , then find $(g \circ f)(5)$

8-

From the following functions , the one-to-one function is .....

(a) $f_1(x) = x + 2$ (b) $f_2(x) = x^2$ (c) $f_3(x) = |x|$ (d) $f_4(x) = 5$

9-

Find the domain of $f : f(x) = \dfrac{3x-5}{x-2}$, then prove that f is one-to-one.

10-

Draw the graph of the function $f : f(x) = \begin{cases} x|x| & \text{when } x < 0 \\ \dfrac{x^4}{|x|} & \text{when } x > 0 \end{cases}$

, then deduce its domain and discuss its type whether it is even , odd or otherwise.

Graphical (basic and piecewise)

11-

Graph the function $f : f(x) = 4 - (x-2)^2$, then deduce its range , its monotony and whether the function is odd , even or otherwise.

12-

Graph the function $f : [-2, 6] \longrightarrow \mathbb{R}$ where $f(x) = \begin{cases} 4 - x & , \quad -2 \le x < 1 \\ x & , \quad 1 \le x \le 6 \end{cases}$

and from the graph deduce its range and discuss its monotonicity.

13-

Graph the function $f : f(x) = \begin{cases} x - 1 & , \quad 2 < x \le 4 \\ -1 & , \quad -2 \le x \le 2 \end{cases}$

from the graph determine its range.

14-

[b] If $f : [-4, 3] \longrightarrow \mathbb{R}$, $f(x) = \begin{cases} 4 & \text{when } x < 0 \\ (x-1)^2 + 1 & \text{when } 0 \le x \le 3 \end{cases}$

, graph the function f , then from the graph , **deduce :**

(**1**) The range.

(**2**) The monotonicity.

(**3**) The type (even , odd , otherwise).

Geometric transformations

15-

Use the graph of the function f where $f(X) = X^2$ to represent the function g where $g(X) = (X-1)^2 + 2$ and from the graph determine the range of the function g and discuss its monotonicity and tell whether it is even, odd or otherwise.

16-

Use the curve of the function f where $f(X) = X^3$ to represent each of the following functions :

(1) $f_1(X) = (X+1)^3$ (2) $f_2(X) = X^3 + 1$

17-

If $f(X) = X^2 - 1$, $g(X) = X + 1$

Graph the function $\frac{f}{g}$, show its domain and its range.

18-

Graph the curve of the function $f : f(X) = X|X|$, then discuss its monotony and its type whether its even, odd or otherwise.

Solving absolute value equations and inqualities

19-

Find in $\mathbb{R}$ the solution set for each of the following :

(1) $|2x-5| \leq 3$

20-

Find in $\mathbb{R}$ the S.S. of the following equation algebraically : $\sqrt{x^2-4x+4}=4$

21-

Find in $\mathbb{R}$ the S.S. of the following inequality algebraically : $|x-3| \geq 5$

22-

Find the solution set in $\mathbb{R}$ of each of the following :

(1) $|2x-3|+|6-4x| \leq 0$ **(2)** $|2x-4|=x+3$

23-

Find graphically in $\mathbb{R}$ the solution set of the inequality : $|5-x|>3$

24-

Find in $\mathbb{R}$ the solution set of the inequality : $\sqrt{4x^2-12x+9}>5$

25-

Find in $\mathbb{R}$ the solution set of each of the following :

(1) $|x+2|+5=9$ **(2)** $|2x-3| \leq 5$

26-

Solve in $\mathbb{R}$ the equation : $2|x-2|-|2-x|=3$

(Unit 2)

Rational exponents and exponintial equations

1-

Find the value of x which satisfies : $2^{x+1} = 25$ by using calculator.

2-

Find in $\mathbb{R}$ the solution set of each of the following :

$$\left(\frac{1}{2}\right)^{x+1} + \left(\frac{1}{2}\right)^{x+3} + \left(\frac{1}{2}\right)^{x+5} = 84$$

3-

Simplify : $\dfrac{16^{x+\frac{1}{4}} \times 9^{x+2}}{8^{x-1} \times 18^{x+2}}$

4-

Find the S.S. of each of the following in $\mathbb{R}$:

$$4^x + 2^{x+1} = 8$$

5-

Put in the simplest form : $\dfrac{9^{4n+1} \times 4^{2-2n}}{3^{9n+1} \times 48^{1-n}}$

6-

If $f(2x) = 3^x$ **, solve in $\mathbb{R}$ the equation :** $f(2x+4) + f(2x) = 90$

Applications

7-

A patient gets 40 mg. of medicine , his body gets rid of 10 % of this medicine every hour.

(1) Write the exponential function which represents the quantity of medicine left after t hours.

(2) Estimate this quantity of medicine left in the body after 4 hours.

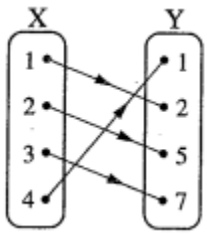
8-

The number of cows in a cattle farm is 80 cows and the reproduction rate of these cows is 18 % annually. Find the number of cows after 4 years , given by $C = a(1 + r)^t$ where t is the number of years , a is the starting amount and r is the annual percentage of reproduction.

The invers function

9-

The opposite figure represents a function $f : X \longrightarrow Y$, then $f^{-1}(2) =$ .....



(a) 5 (b) 1

(c) 3 (d) 4

10-

If $f(x) = 3 + \sqrt{2x-1}$, **find :** $f^{-1}(x)$ and find the domain of f^{-1} and its range.

11-

If $f(x) = x - 3$, then find the inverse function of f

12-

Find the inverse function of f where $f(x) = x^3 + 1$

13-

If $f(x) = 5x$, find $f^{-1}(x)$ and represent it graphically.

Logarithmic function and its properties

14-

Without using calculator , find the value of : $\log 25 + \dfrac{\log 8 \times \log 16}{\log 64}$

15-

Find in $\mathbb{R}$ the S.S of the equation :

$\log_7 x + \log_7 (x + 6) = 1$

16-

If $x\,y = 16$ **, prove that :** $3 \log_2 x + 4 \log_2 y - \log_2 x\,y^2 = 8$

17-

Find in $\mathbb{R}$ the solution set of each of the following :

$$\log_5 (x^2 - 25) - \log_5 (x - 5) = 2$$

18-

Find the S.S. in $\mathbb{R}$ of the equation :

$(\log_3 x)^2 - 2 \log_3 x - 3 = 0$

19-

Simplify : $(1)\ \dfrac{1}{\log_a ab} + \dfrac{1}{\log_b ab}$

20-

Find the S.S. of the following equation in $\mathbb{R}$: $\log_2 x + \log_2 (x + 1) = 1$

21-

Without using calculator , find the value of the following :

$2 \log 25 + \log \left(\frac{1}{3} + \frac{1}{5}\right) + 2 \log 3 - \log 30$

22-

Solve the equation in $\mathbb{R}$: $\log_2 x + \log_2 (x + 1) = 1$

23-

Solve in $\mathbb{R}$ the following equation : $\log_4 x = 1 - \log_4 (x - 3)$

24-

Find in $\mathbb{R}$ the solution set for each of the following :

$\log_3 x + \log_x 3 = 2$

25-

(1) Find in $\mathbb{R}$ the solution set of the equation : $\log_3 x + \log_x 3 = 2$

(2) Prove that : $\frac{\log 729 - \log 64}{\log 9 - \log 4} = 3$

26-

Prove that : $\log_2 \frac{4}{11} - \log_2 \frac{7}{130} + \log_2 \frac{77}{65} = \log_5 125$

27-

Find in $\mathbb{R}$ the S.S. of the equation :

$\log_4 x = 1 - \log_4 (x - 3)$

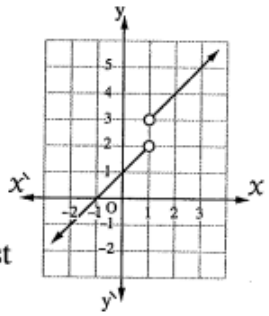
28-

Prove that : $\log_b a \times \log_c b \times \log_d c \times \log_a d = 1$

(Unit 3) Introduction of limits of functions

1-

Choose the correct answer from those given :

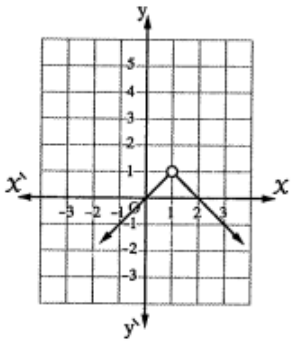


(**1**) The opposite figure represents the graph of the function f , then

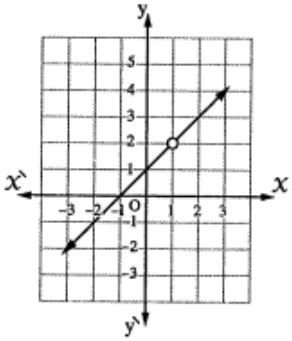
$\lim_{x \to 1} f(x) =$ .....

(a) 2 (b) 3 (c) 1 (d) not exist

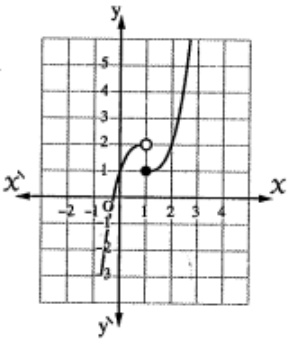
(**2**) Which of the following functions has no limit at $x = 1$?



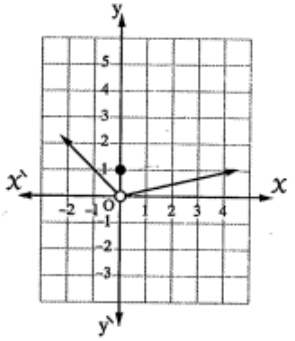
(a)



(b)



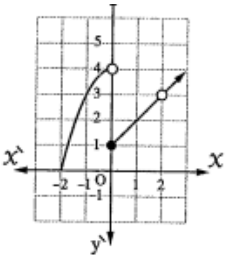
(c)



(d)

2-

In the opposite figure , find :



(**1**) f (zero$^+$) (**2**) f (zero$^-$)

(**3**) f (2) (**4**) $\lim_{x \to 2} f(x)$

Finding tke limit of the function algebrically

3-

Choose the correct answer from those given :

(1) $\lim_{x \to 0} \frac{1+x}{4x-1} =$ .....

(a) -1 (b) $\frac{1}{4}$ (c) $-\frac{1}{4}$ (d) 1

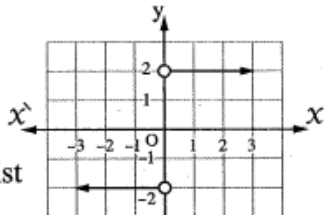
(2) $\lim_{x \to 3} \frac{x^2-9}{x-3} =$ .....

(a) -6 (b) zero (c) 3 (d) 6

(3) The opposite figure represents $f(x)$

Then : $\lim_{x \to 2} f(x) =$ .....

(a) 0 (b) -2 (c) 2 (d) not exist



4-

Find :

$$\lim_{x \to 3} \frac{x^2-x-6}{x-3}$$

5-

Find :

$$\lim_{x \to -1} \left(\frac{5x^2+5x}{3x^2-3}\right)$$

6-

$$\lim_{x \to 0} \frac{x^2+x}{x} = \text{.....}$$

7-

Find : $\lim_{x \to 1} \frac{x^3-2x^2+1}{x^2+x-2}$

8-

Find : **(2)** $\lim_{x \to 2} \frac{x^2 - 5x + 6}{x^3 - 8}$

9-

Find : $\lim_{x \to 5} \frac{x - 5}{2x - 3}$

10-

Find : $\lim_{x \to 2} \frac{x^3 + 8}{x + 2}$

11-

$\lim_{x \to -3} \frac{x^2 + 4x + 3}{x^2 - 9}$

12-

Find : (1) $\lim_{x \to -2} \frac{x^3 + 8}{x + 2}$

The law

13-

$$\lim_{x \to 4} \frac{\sqrt{x-3}-1}{x-4}$$

14-

Find : (1) $\lim_{x \to -2} \frac{(x+3)^5-1}{x+2}$

15-

Find : (1) $\lim_{x \to 2} \frac{x^3+8}{x+2}$

16-

Find : (1) $\lim_{x \to 3} \frac{x^5-243}{x^2-9}$

17-

Find : $\lim_{x \to 5} \frac{(x-3)^5-32}{x-5}$

18-

Find : (1) $\lim_{x \to -3} \frac{x^4-81}{x^5+243}$

19-

Find : (1) $\lim_{x \to -2} \frac{\sqrt[3]{x} + \sqrt[3]{2}}{x+2}$

20-

Find : (1) $\lim_{x \to \sqrt{5}} \frac{x^7 - 125\sqrt{5}}{x^4 - 25}$

21-

$\lim_{x \to 5} \frac{(x-3)^5 - 32}{x^2 - 5x}$

22-

$\lim_{x \to -3} \frac{x^4 - 81}{x^5 + 243}$

23-

$\lim_{x \to -3} \frac{x^4 - 81}{x^5 + 243}$

24-

$\lim_{x \to 2} \frac{x^7 - 128}{x^5 - 32}$

25-

Find : (1) $\lim_{x \to 2} \frac{2x^4 - 32}{x^2 - 4}$

26-

Find : (1) $\lim_{x \to \frac{1}{3}} \frac{27x^4 - \frac{1}{3}}{3x - 1}$

Limit of the function at infinity

26-

Find : (1) $\lim_{x \to \infty} \frac{2x^3 - 5x}{x^4 + 3}$

27-

$\lim_{x \to \infty} \frac{2x^3 - 9}{|3x|^3 + 7}$

28-

$\lim_{x \to \infty} \frac{5x^{-3} + 4x^{-2} - 3}{7x^{-3} - 2x^{-2} + 8}$

29-

Find : (1) $\lim_{x \to \infty} \left(\sqrt{4x^2 - 2x + 1} - 2x\right)$

30-

$\lim_{x \to \infty} \frac{2x^2 - x + 1}{x^3 - x^2 + 1}$

31-

Find : (1) $\lim_{x \to \infty} \frac{2x^3 - 3}{3x^2 + 1}$

32-

$$\lim_{x \to \infty} \frac{x^{-2}+3}{x^{-3}+6}$$

33-

Find : $(1)\ \lim_{x \to \infty} \frac{\sqrt{9x^2+3}}{6x-1}$

34-

$$\lim_{x \to \infty} \frac{5-6x-3x^2}{2x^2+x+4}$$

35-

$$\lim_{x \to \infty} \frac{-x}{\sqrt{4+x^2}}$$

36-

Find : $(1)\ \lim_{x \to \infty} \frac{x^3-4x+5}{(2x-1)^3}$

37-

Find : $(1)\ \lim_{x \to \infty} \frac{5-6x-3x^2}{2x^2+x+4}$

38-

Find : $(1)\ \lim_{x \to \infty} \frac{3x^2+x-1}{8x^2-3}$

39-

$$\lim_{x \to \infty} \frac{2x^{-1} - 3x^{-2}}{4 + x^{-1}}$$

40-

$$\lim_{x \to \infty} \frac{3x + \sqrt{4x^2 + 5}}{5x - 3}$$

41-

$$\lim_{x \to \infty} \left(\sqrt{x^2 + 5x} - x\right)$$

42-

$$\lim_{x \to \infty} \frac{x^3 - 2}{|x|^3 + 1}$$

43-

Find : (1) $\lim_{x \to \infty} \frac{x^3 - 4x + 5}{(2x - 1)^3}$

44-

Find : (1) $\lim_{x \to \infty} (\sqrt{x^2 + 5x} - x)$

45-

$$\lim_{x \to \infty} \frac{1}{x}\sqrt{3+4x^2}$$

Limits of tragnometric functions

46-

$$\lim_{x \to 0} \frac{1 - \cos^2 2x}{3x^2}$$

47-

$$\lim_{x \to 0} \frac{x - x\cos x}{\sin^2 3x}$$

48-

$$\lim_{x \to 0} \frac{\sin 3x - \sin 2x}{5x}$$

49-

Find : (1) $\lim_{x \to 0} \frac{2x + \sin 3x}{\tan 5x}$

50-

Find : $\lim_{x \to 0} \frac{x^3 + \sin 3x}{5x \cos 2x}$

51-

Find : (1) $\lim_{x \to 0} \frac{x - x\cos x}{\sin^2 3x}$

52-

$$\lim_{x \to 0} \frac{x^2 + \sin 3x}{5x \cos 2x}$$

53-

$$\lim_{x \to 0} \frac{\sin^2 3x - \tan 2x^2}{x^2 \cos 4x}$$

54-

$$\lim_{x \to 0} \frac{x \tan 2x \cos 3x}{x^2 + \sin^2 3x}$$

55-

Find : $(1) \lim_{x \to 1} \frac{\sin(x-1)}{x^2 + x - 2}$

56-

Find : $(1) \lim_{x \to 0} \frac{x - x \cos x}{\sin^2 3x}$

57-

Find : $(1) \lim_{x \to 0} \frac{1 - \cos x + \sin x}{1 - \cos x - \sin x}$

58-

$$\lim_{x \to 0} \frac{\sin 3x - \tan 2x}{5x}$$

59-

$$\lim_{x \to 0} \frac{\sin 2x + 5 \sin 3x}{x}$$

60-

$$\lim_{x \to 0} \frac{x \tan x + \sin^2 3x}{2x^2 + \sin 3x^2}$$

Existence of the limit of a piecewise function

61-

Find : (1) $\lim_{x \to \pi} f(x)$ where $f(x) = \begin{cases} \frac{2 \sin x}{\pi - x} & , \quad x < \pi \\ 1 - \cos x & , \quad x > \pi \end{cases}$

(2) $\lim_{x \to \infty} x \tan x^{-1}$

62-

$$f(x) = \begin{cases} \frac{x \tan x + \sin^2 3x}{5x^2} \\ 2 \cos 2x \end{cases}$$

63-

Discuss the existence of the limit of the function f where

$$f(x) = \begin{cases} \frac{1 - \cos x}{x} & , \quad x > 0 \\ 2 \sin x & , \quad x \leq 0 \end{cases} \quad \text{at } x = 0$$

64-

Find : $\lim_{x \to 3} f(x)$ if $f(x) = \begin{cases} x^2 + 1 & , \quad x < 3 \\ 3x + 1 & , \quad x > 3 \end{cases}$

65-

If $f(x)=\begin{cases} x^2-2x & , \quad -2<x<1 \\ 3x-4 & , \quad 1\le x<4 \end{cases}$

, discuss the existence of each of the following :

(1) $\lim\limits_{x\longrightarrow -2} f(x)$ **(2)** $\lim\limits_{x\longrightarrow 1} f(x)$

66-

If $\lim\limits_{x\longrightarrow 2} f(x)=7$ where $f(x)=\begin{cases} x^2+3\,m & , \quad x<2 \\ 5x+k & , \quad x>2 \end{cases}$, find m and k

67-

Discuss the existence of Lim $f(x)$ which $x\longrightarrow 0$ where

$$f(x)=\begin{cases} \dfrac{5x^2+\tan 2x^2}{\sin^2 x} & , \quad x>0 \\ 7\cos 3x & , \quad x<0 \end{cases}$$

68-

Find : **(1)** $\lim\limits_{x\longrightarrow \pi} f(x)$ where $f(x)=\begin{cases} \dfrac{2\sin x}{\pi-x} & , \quad x<\pi \\ 1-\cos x & , \quad x>\pi \end{cases}$

(2) $\lim\limits_{x\longrightarrow \infty} x\tan x^{-1}$

69-

If $f(x) = \begin{cases} \dfrac{x^2 - 7x + 12}{x - 3} & , \quad x > 3 \\ a\,x - 7 & , \quad x < 3 \end{cases}$

, $\lim_{x \longrightarrow 3} f(x) = -1$, then find the value of a

Continuity

70-

Discuss the continuity of the function f

where $f(x) = \begin{cases} 8 - x & , \quad x \geq 3 \\ x + 2 & , \quad x > 3 \end{cases}$ at $x = 3$

71-

Discuss the continuity of the function f where :

$$f(x) = \begin{cases} \dfrac{x^2 + x - 2}{x + 2} & , \quad x > -2 \\ 3x + 5 & , \quad x \leq -2 \end{cases} \quad \text{at } x = -2$$

72-

Discuss the continuity of the function f on its domain where :

$$f(x) = \begin{cases} 1 + \sin x & , \quad \text{when } 0 \leq x \leq \dfrac{\pi}{2} \\ 1 - \cos 2x & , \quad \text{when } x > \dfrac{\pi}{2} \end{cases}$$

73-

Discuss the continuity of the function f :

$$f(x) = \begin{cases} \dfrac{\sin(x-2)}{x^2 - 4} & , \quad x < 2 \\ 1 - \dfrac{3}{x^2} & , \quad x \geq 2 \end{cases} \quad \text{at } x = 2$$

74-

Discuss the continuity of f : $f(x) = \begin{cases} \dfrac{x^2 - x - 6}{x - 3} & , \quad x < 3 \\ 2x - 1 & , \quad x \geq 3 \end{cases}$ at $x = 3$

75-

Discuss the continuity of the function f where

$$f(x) = \begin{cases} \dfrac{\sin x}{2x - 2\pi} & , \quad x < \pi \\ \dfrac{1}{2}\cos x & , \quad x \geq \pi \end{cases} \quad \text{at } x = \pi$$

76-

Discuss the continuity of f at $x = 1$

where $f(x) = \begin{cases} x^2 + 3 & , \quad x \geq 1 \\ \dfrac{x^2 + 2x - 3}{x - 1} & , \quad x < 1 \end{cases}$

77-

Find the value of a which makes the function f

where $f(x) = \begin{cases} \dfrac{x^2 - 5x + 6}{x^3 - 8} & , \quad x \neq 2 \\ \dfrac{-2}{|a|} & , \quad x = 2 \end{cases}$ continuous at $x = 2$

78-

Find the value of k which makes the function f continuous at $x = 2$ where

$$f(x) = \begin{cases} \dfrac{x^2 + x - 6}{x^3 - 8} & , \quad x \neq 2 \\ \dfrac{2}{|k|} & , \quad x = 2 \end{cases}$$

79-

Discuss the continuity of f : $f(x) = \begin{cases} x^2 + 3 & , \quad x \geq 1 \\ \dfrac{x^2 + 2x - 3}{x - 1} & , \quad x < 1 \end{cases}$ at $x = 1$

80-

Discuss the continuity of the function f

where $f(x) = \begin{cases} x^2 + 3 & , \quad x \geq 1 \\ \dfrac{x^2 + 2x - 3}{x - 1} & , \quad x < 1 \end{cases}$ at $x = 1$

81-

Find the value of the constant a if the function f :

where $f(x) = \begin{cases} \dfrac{(x+3)^4 - 81}{x} & , \quad x \neq 3 \\ a & , \quad x = 3 \end{cases}$ is continuous at $x = 3$

82-

Discuss the continuity of the function f where :

$$f(x)=\begin{cases}\dfrac{\sin x}{\pi-x} & , \quad x\neq\pi \\ 1 & , \quad x=\pi\end{cases} \quad \text{at } x=\pi$$

(Unit 4) The sine rule

1-

Solve Δ ABC in which m (∠ B) = 35° , m (∠ C) = 70° , and the diameter length of its circumcircle = 32 cm.

2-

In Δ ABC , if m (∠ A) = 35° , a = 17 cm. and b = 20 cm.
Prove that : Δ ABC has two solutions , then find them.

3-

Find the perimeter of Δ ABC in which m (∠ A) = 57° 13̀ , c = 8.7 cm.
and m (∠ B) = 64° 18̀

4-

ABC is a triangle in which : m (∠ A) = 35° , a = 8 cm. and b = 6 cm. **Find :** m (∠ B)

5-

ABC is a triangle in which : b = 12 cm. , m (∠ B) = 75° and m (∠ C) = 45° **Find :**

(1) a

(2) The area of Δ ABC

(3) The radius length of the circumcircle of the triangle ABC

6-

ABC is a triangle in which m (∠ A) : m (∠ B) : m (∠ C) = 3 : 4 : 3

If a = 5 cm. , find the perimeter of Δ ABC

7-

Solve the triangle ABC in which a = 8 cm. , m (∠ A) = 60° and m (∠ B) = 40°

8-

Solve the acute-angled triangle ABC in which a = 21 cm. , b = 25 cm. and the diameter length of its circumcircle = 28 cm.

9-

Find the shortest side length in Δ ABC , in which : m (∠ A) = 43° , m (∠ B) = 70° and c = 9 cm. Find the area of the triangle ABC.

10-

ABC is a triangle in which : AC = 4.7 cm. , m (∠ B) = 34° and m (∠ C) = 66°

Find the length of $\overline{BC}$, then find the area of its circumcircle.

11-

ABC is a triangle in which m (∠ A) = 40° , a = 5 cm. and b = 7 cm.

Find m (∠ B) approximating to the nearest minute.

12-

Solve the triangle ABC in which a = 5 cm. , b = 7 cm. and m (∠ C) = 65°

13-

ABC is a triangle in which m (∠ A) = 85° , m (∠ B) = 55° and c = 5 cm.
Find the area of the circumcircle of Δ ABC

The cosine rule

14-

The perimeter of the triangle ABC is 52 cm. , a = 13 cm. and b = 17 cm.
Find the measure of the greatest angle.

15-

Solve the triangle ABC in which a = 5 cm. , b = 7 cm. and m (∠ C) = 65°

16-

Solve the triangle ABC in which a = 9 cm. , b = 7 cm. and c = 5 cm. , then find its area.

17-

In Δ ABC , if a = 4 cm. , b = 5 cm. and c = 6 cm. , **prove that :** cos C = cos 2 A
, then find the circumference of the circumcircle of Δ ABC

18-

Solve the triangle ABC in which a = 9 cm. , b = 15 cm. and m (∠ C) = 106°

19-

Solve the triangle ABC in which a = 15 cm. , b = 13 cm. and c = 14 cm.

20-

ABC is a triangle in which a = 27 cm. , b = 35 cm. and c = 18 cm.

Find the measure of the greatest angle.

Solution of the triangle

21-

In ΔABC , $\cos A = \frac{2}{5}$, $b = 2.5$ cm. and $c = 2$ cm.

Prove that : ΔABC is an isosceles triangle and find its area.

22-

In ΔABC , if $m(\angle A) = 35°$, $a = 17$ cm. and $b = 20$ cm.

Prove that : ΔABC has two solutions , then find them.

23-

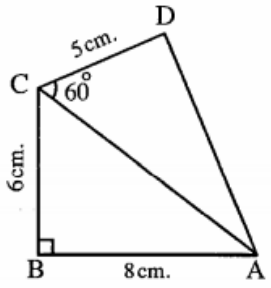
In the opposite figure :

ABCD is a quadrilateral in which

AB = 8 cm. , BC = 6 cm. , $m(\angle B) = 90°$

, DC = 5 cm. and $m(\angle ACD) = 60°$

Find the area of the circumcircle of ΔADC



24-

In ΔABC , show whether it has only 1 solution , 2 solutions or non , given your answer to the nearest decimal :

(1) $m(\angle B) = 110°$, $b = 8$ cm. and $c = 5$ cm.

(2) $m(\angle A) = 60°$, $a = 7$ cm. and $b = 9$ cm.

25-

Show whether the triangle ABC has one , two or no solution ,

given that : m ($\angle$ A) = 100° , a = 12 cm. and b = 15 cm.

26-

ABC is a triangle in which m ($\angle$ A) = 52° , a = 21 cm. and b = 26 cm.

Prove that : Δ ABC has two solutions , then find them.

27-

ABC is a triangle in which : $\frac{\sin A}{3} = \frac{\sin B}{4} = \frac{\sin C}{5}$

and its perimeter = 24 cm. Find its area.